C.U.SHAH UNIVERSITY
Winter Examination-2015

Subject Name : Advance Mathematics

Subject Code : 2TE02AMT2   **Branch :** Diploma(All)

Semester : 02   Date : 19/11/2015   Time :10:30 am To 1:30 pm   **Marks :**70

Instructions:
- (1) Use of Programmable calculator & any other electronic instrument is prohibited.
- (2) Instructions written on main answer book are strictly to be obeyed.
- (3) Draw neat diagrams and figures (if necessary) at right places.
- (4) Assume suitable data if needed.

Q-1   **Attempt the following questions:**   **(14)**

a)   Magnitude of $3i - 4j - 5k$ is _____.

(a) $2\sqrt{5}$ (b) 0 (c) -6 (d) $5\sqrt{2}$

b)   If $a = i + j$ and $b = j - k$ then $a \cdot b = $ _____.

(a) 0 (b) -1 (c) 1 (d) none of these

c)   If $a = i - j + k$ then $\hat{a} = $ _____.

(a) $\dfrac{1}{\sqrt{2}}(1, -1, 1)$ (b) $\dfrac{1}{\sqrt{3}}(1, -1, 1)$ (c) $\sqrt{3}$ (d) none of these

d) $\lim\limits_{n \to \infty}\left(1 + \dfrac{1}{n}\right)^{4n} = $ _____

(a) e^4 (b) e (c) 1 (d) none of these

e) $\lim\limits_{x \to 0}\dfrac{\sin\sqrt{x}}{\sqrt{x}} = $ _____

(a) 0 (b) e (c) $\dfrac{1}{2}$ (d) 1

f) $\lim\limits_{x \to 0}\dfrac{5^x - 1}{x} = $ _____

(a) 0 (b) e^5 (c) $\log_e 5$ (d) 1

g)   If $f(x) = x^2 - 1$ then $f(-1) = $ _____.

(a) -2 (b) 0 (c) -1 (d) none of these

h) $\dfrac{d(6^x)}{dx} = $ _____

(a) $6^x\log_e 6$ (b) $\log_e 6$ (c) $x^6\log_e 6$ (d) $6^x\log_e x$

i) $\dfrac{d\left(\sqrt{x}\right)}{dx} = \underline{\hspace{1cm}}$

    (a) $2x^{\frac{1}{2}}$ (b) $\dfrac{1}{2}\sqrt{x}$ (c) $\dfrac{2}{\sqrt{x}}$ (d) $\dfrac{1}{2\sqrt{x}}$

j) $\dfrac{d\left(x^{e}\right)}{dx} = \underline{\hspace{1cm}}$

    (a) x^{e-1} (b) xe^{x-1} (c) ex^{e-1} (d) e^{x-1}

k) $\int e^{x}\,dx = \underline{\hspace{1cm}}$

    (a) $\log x$ (b) e^{x} (c) $\dfrac{e^{x}}{\log x}$ (d) none of these

l) $\int \dfrac{1}{x^{2}+1}\,dx = \underline{\hspace{1cm}}$

    (a) $\tan^{-1}x+c$ (b) $\cos^{-1}x+c$ (c) $\cot^{-1}x+c$ (d) $\sin^{-1}x+c$

m) $\int \dfrac{1}{\sqrt{1-x^{2}}}\,dx = \underline{\hspace{1cm}}$

    (a) $\tan^{-1}x+c$ (b) $\cos^{-1}x+c$ (c) $\cot^{-1}x+c$ (d) $\sin^{-1}x+c$

n) $\int_{0}^{1} x^{2}\,dx = \underline{\hspace{1cm}}$

    (a) 2 (b) 1 (c) $\dfrac{1}{3}$ (d) none of these

Attempt any four questions from Q-2 to Q-8

Q-2   **Attempt all questions**   **(14)**

a) If $a = i + 2j - k$, $b = 3i + j + 2k$ and $c = -2i - j + 5k$ then find $|2a + 3b - c|$.   **(5)**

b) If $a = 2i - j$, $b = i + 3j - 2k$ then obtain $|(a + b)\times(a - b)|$.   **(5)**

c) For what value of p, vectors $2i + 3j - k$ and $pi - j + 3k$ are perpendicular to each other?   **(4)**

Q-3   **Attempt all questions**   **(14)**

a) A particle moves from the point $3i - 2j + k$ to the point $i + 3j - 4k$ under the effect of constant forces $i - j + k$, $i + j - 3k$ and $4i + 5j - 6k$. Find the work done.   **(5)**

b) Prove that angle between two vectors $i + 2j$ and $i + j + 3k$ is $\sin^{-1}\left(\sqrt{\dfrac{46}{55}}\right)$.   **(5)**

c) Find unit vector which is perpendicular to $a = 5i + 7j - 2k$ and $b = 3i + j - 2k$.   **(4)**

Q-4   **Attempt all questions**   **(14)**

a) Prove that $\lim\limits_{x \to 3} \dfrac{\sqrt{x+2}-\sqrt{5}}{\sqrt{x+4}-\sqrt{7}} = \dfrac{\sqrt{35}}{5}$ **(5)**

b) Evaluate: $\lim\limits_{\theta \to 0} \dfrac{\operatorname{cosec}\theta - \cot\theta}{\theta}$ **(5)**

c) Prove that If $f(x) = \log\left(\dfrac{x-1}{x}\right)$ then prove that $f(x) + f(-x) = f(x^2)$. **(4)**

Q-5   **Attempt all questions** **(14)**

a) Find derivative of $f(x) = x^3$ using definition. **(5)**

b) Find $\dfrac{dy}{dx}$ if $y = \log\left[x + \sqrt{x^2 + a^2}\right]$. **(5)**

c) Find $\dfrac{dy}{dx}$ if $y = \dfrac{a + b\sin x}{a\sin x + b}$. **(4)**

Q-6   **Attempt all questions** **(14)**

a) Find $\dfrac{dy}{dx}$ if $e^x + e^y = e^{x+y}$. **(5)**

b) The equation of motion of a particle is $S = 2t^3 + 3t^2 - 12t + 5$.
Find velocity at $t = 0$ and acceleration at $t = 1$. **(5)**

c) Find $\dfrac{dy}{dx}$ if $y = (\sin x)^{\tan x}$. **(4)**

Q-7   **Attempt all questions** **(14)**

a) Evaluate: $\displaystyle\int \dfrac{(1-3x)^2}{x^3}\,dx$ **(5)**

b) Evaluate: $\displaystyle\int x \log x\,dx$ **(5)**

c) Evaluate: $\displaystyle\int \sin^4 x \cos x\,dx$ **(4)**

Q-8   **Attempt all questions** **(14)**

a) Prove that $\displaystyle\int_{0}^{\frac{\pi}{2}} \dfrac{\sec x}{\sec x + \cosec x}\,dx = \dfrac{\pi}{4}$ **(7)**

b) Find the area of standard ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$ **(7)**

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